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An ILS algorithm for the bin packing problem with generalized precedence constraints

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Outline

1 Introduction

2 Mathematical Formulation

3 Proposed Algorithm

4 Computational Results

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Bin Packing Problem (BPP)

- Classical combinatorial optimization problem
- $N = \{1, 2, \ldots, n\}$ – set of items
  - Each item $j \in N$ has weight w_j
- $M = \{1, 2, \ldots, m\}$ – set of bins
  - Each bin $i \in M$ has a homogeneous capacity C
- Objetive: Pack all items, minimizing the number of bins.
- Constraints: Do not exceed the bin capacity.
- $\mathcal{NP}$-Hard.

- $N = \{1, 2, \ldots, n\}$ – set of items
  - Each item $j \in N$ has weight w_j
  - Set of precedence relationship between pairs of items: $t_{jk} = \tau$ (i.e., item k should be *packed at least* $\tau \in \{0, 1, \ldots, T\}$ *bins* after item j)
- $M = \{1, 2, \ldots, m\}$ – set of bins
  - Each bin $i \in M$ has a homogeneous capacity C
- Objetive: Pack all items, minimizing the number of bins.
- Constraints: Do not exceed the bin capacity and respect the precedence relationship between items.
- $\mathcal{NP}$-Hard.

BPP with Generalized Precedence Constraints (BPPGP)

- Generalizes
  - BPP with precedence constraints (BPP-P): $t_{jk} = 1$
    - i.e., item k should be *packed at least one bin* after item j
  - Simple Assembly Line Balancing Problem Type 1 (SALBP-I): $t_{jk} = 0$
    - i.e., task k should be *processed* after task j

BPPGP

$$t_{jk} \in \{0, 1, \ldots, T\}$$

Example of a BPPGP instance

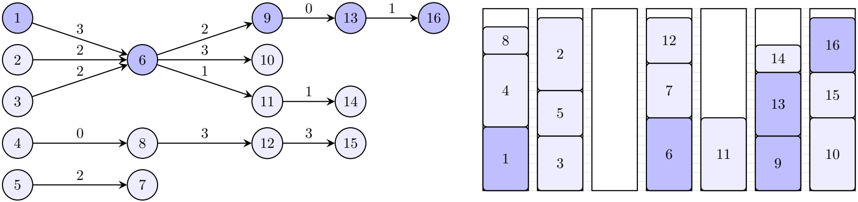

Figure : Precedence graph and an optimal solution (7 bins) for a BPPGP instance with 16 items.

Applications

- Assembling lines with proof tests/inspection;
- TV advertisement: repeat the advertisement after at least τ commercial breaks from the last appearance;
- Civil construction: scheduling activities (concrete components require some days to get resistance);

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BPPGP - Definition and Notation

- Sets:
  - $N = \{1, 2, \dots, n\}$ - set of items;
  - $M = \{1, 2, \dots, m\}$ - set of bins;
  - $P(k)$ - set of preceding items of $k \in N$;
  - $A = \{(j, k) : k \in N, j \in P(k)\}$ - set of arcs (precedences)
- Input data:
  - C - bin capacity;
  - w_j - weight of item $j \in N$;
  - t_{jk} - precedence value between items j and k, $(j, k) \in A$;
- Binary variables:
  - $y_i = \begin{cases} 1, \text{ if bin } i \in M \text{ is used} \\ 0, \text{ if not.} \end{cases}$
  - $x_{ij} = \begin{cases} 1, \text{ if item } j \in N \text{ is assigned to bin } i \in M \\ 0, \text{ if not.} \end{cases}$

BPPGP - Mathematical Formulation

$$\min \quad \sum_{i \in M} y_i \tag{1}$$

$$s.t. \quad \sum_{i \in M} x_{ij} = 1 \qquad j \in N, \tag{2}$$

$$\sum_{j \in N} w_j x_{ij} \leq C y_i \qquad i \in M, \tag{3}$$

$$\sum_{i \in M} i x_{ik} \geq \sum_{i \in M} i x_{ij} + t_{jk} \qquad (j,k) \in A, \tag{4}$$

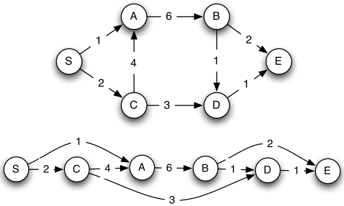
$$y_i \geq y_{i+1} \qquad i = 1, \ldots, m-1, \tag{5}$$

$$y_i \in \{0,1\} \qquad i \in M, \tag{6}$$

$$x_{ij} \in \{0,1\} \qquad i \in M, j \in N. \tag{7}$$

Computing heads and tails

- $head_j$ - the minimum number of bins before item $j \in N$;
- $tail_j$ - the minimum number of bins after item $j \in N$.
  - Adding two dummy nodes 0 and $n + 1$
  - $head_j$ - longest path from 0 to j
  - $tail_j$ - longest path from j to $n + 1$
  - Dynamic programming



BPPGP - Mathematical Formulation

$$
\begin{aligned}
\min \quad & \sum_{i \in M} y_i & & \\
s.t. \quad & \sum_{i \in M} x_{ij} = 1 & & j \in N, \\
& \sum_{j \in N} w_j x_{ij} \leq C y_i & & i \in M, \\
& \sum_{i \in M} i x_{ik} \geq \sum_{i \in M} i x_{ij} + t_{jk} & & (j,k) \in A, \\
& y_i \geq y_{i+1} & & i = 1, \ldots, m-1, \\
& x_{ij} = 0 & & i \in M, j \in N : i \leq head_j, & (8) \\
& x_{ij} = 0 & & i \in M, j \in N : i > m - tail_j, & (9) \\
& y_i \in \{0,1\} & & i \in M, \\
& x_{ij} \in \{0,1\} & & i \in M, j \in N.
\end{aligned}
$$

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The Proposed ILS Algorithm

- Composed by 5 main procedures:

Algorithm ILS

```
preprocessing()
bestLB ← computeLowerBounds()
bestSol ← sol ← generateInitialSolution()
Iter ← 0
while bestSol > bestLB and time limit is not reached do
    sol ← localSearch(sol)
    if sol < bestSol then
        bestSol ← sol
    else
        sol ← bestSol
    sol ← perturbation(sol)
return bestSol;
```

Preprocessing procedures

- Lift the weights and precedence values
  - Subset sum with conflicts
  - *BPP linear relaxation*

Lower Bounds

- $LB_1 = \lceil \sum_{j=1}^{n} w_j / C \rceil$.
- LB_2: Adding two dummy nodes, 0 and $n+1$, to the precedence graph (with precedence values equal to 0), it computes the longest path from 0 to $n+1$ (+1).

Lower Bounds

- P: subset of items in the longest path from 0 to $n+1$
- B^P: set of bins required to pack the items in P
- S': maximum (fractional) weight of items $j \in N \setminus P$ that can be packed in the (remaining capacity of) bins of B^P
  - *Max-flow procedure* from $N \setminus P$ to B^P

$$LB_3 = LB_2 + \left\lceil \frac{\sum_{j \in N \setminus P} w_j - S'}{C} \right\rceil$$

Lower Bounds

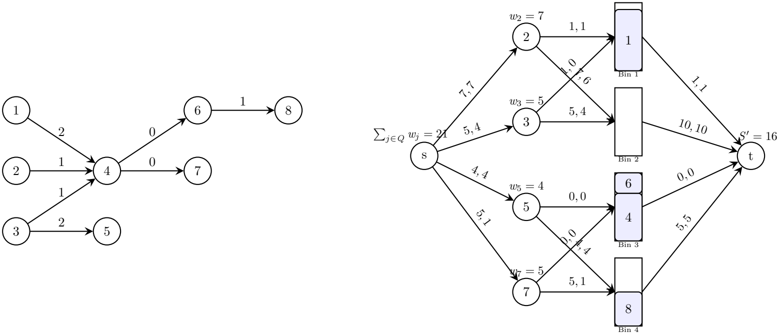


Figure : Example of LB_3.

$$LB_3 = LB_2 + \left\lceil \frac{\sum_{j \in Q} w_j - S'}{C} \right\rceil = 4 + \left\lceil \frac{21 - 16}{10} \right\rceil = 5$$

Lower Bounds

- $I_{r,q}$: subset of items $j \in N$ such that $head_j \geq r$ and $tail_j \geq q$
- $LB_\alpha(I_{r,q})$: any valid BPPGP lower bound for $I_{r,q}$

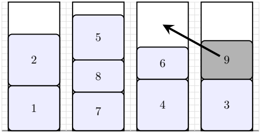
$$LB_4(LB_\alpha) = \max_{\substack{r=0,\ldots,LB_2-1 \\ q=0,\ldots,LB_2-r-1}} \{r + LB_\alpha(I_{r,q}) + q\}$$

$$LB_{BPPGP} = \max\{LB_1, LB_2, LB_3, LB_4\}$$

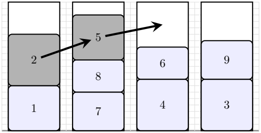
ILS: Constructive procedures

- First Fit Algorithm: insert an item in the first feasible bin.
- Subset Sum procedure: one bin at a time, maximizing the load (sum of item weights).

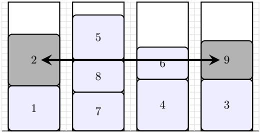
- 4 Operators:
  - Swap(1,1), Swap(2,1), Relocate and Push
- Objective: Minimize the minimum load
  - Removing empty bins



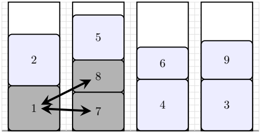
Relocate



Push



Swap(1,1)



Swap(2,1)

ILS: Local Search

- Promising moves are stores in a pool
  - Executed from the best to the worst
  - Checking if it still leads to an improvement ($\mathcal{O}(1)$)
  - Improvement of the convergence

ILS: Perturbation

- Remove k random items, $k \in [K_{min}, K_{max}]$.
- Remove the empty bins (if possible)
- Reinsert items using *First Fit* algorithm (random order)
  - In case of infeasibility, repeat for $k = k - 1$;

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Computational Environment

- C++ programming language.
- Xeon 2.4 GHz, 24 GB RAM.
- Linux Ubuntu 14.04 LTS.
- MIP Solver CPLEX 12.6.

Instances

- *Otto et al. (2011)* for SALBP-I (and BPP-P)
  - Capacity, item weights and precedences
  - 4 groups: 20, 50, 100, and 1000 items
  - 525 instances for each group
- BPPGP-01
  - Arcs weights randomly generated between 0 and 1
- BPPGP-03
  - Arcs weights randomly generated between 0 and 3

Parameters

- ILS & CPLEX:
  - One thread
  - Time Limit = 300 seconds
- Perturbation:
  - $k \in [1, 7]$, if $n \leq 100$;
  - $k \in [1, 50]$, if $n = 1000$;

Computational Results - SALBP-I

Table : Results for the SALBP-I on 100-items instances. Deviation to the best known lower bound.

Instance	Structure	OS	Distribution	BLB*	BUB	Beam-ACO	SALOME	Tabu-Simple		t-bounded-DP		BBR	**ILS**
						180sec	180sec	180sec	3600sec	180sec	3600sec	3600sec	300sec
1-25	BN	0.2	bimodal	22.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
26-50	BN	0.2	bottom	14.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
51-75	BN	0.2	middle	53.56	0.08	0.60	0.92	0.16	0.12	0.48	0.40	0.12	0.12
76-100	BN	0.6	bimodal	22.56	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00
101-125	BN	0.6	bottom	14.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
126-150	BN	0.6	middle	53.56	0.08	0.68	0.76	0.12	0.08	0.08	0.08	0.08	0.12
151-175	CH	0.2	bimodal	22.88	0.00	0.04	0.00	0.08	0.08	0.00	0.00	0.00	0.08
176-200	CH	0.2	bottom	14.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
201-225	CH	0.2	middle	52.44	0.08	0.52	1.16	0.20	0.16	0.52	0.32	0.16	0.20
226-250	CH	0.6	bimodal	22.76	0.00	0.08	0.00	0.12	0.08	0.00	0.00	0.00	0.12
251-275	CH	0.6	bottom	13.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
276-300	CH	0.6	middle	55.84	0.00	0.84	0.48	0.16	0.08	0.04	0.00	0.00	0.28
301-325	MIX	0.2	bimodal	22.76	0.00	0.00	0.00	0.08	0.04	0.00	0.00	0.00	0.04
326-350	MIX	0.2	bottom	14.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
351-375	MIX	0.2	middle	53.96	0.04	0.36	0.80	0.04	0.04	0.28	0.16	0.04	0.04
376-400	MIX	0.6	bimodal	22.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
401-425	MIX	0.6	bottom	14.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
426-450	MIX	0.6	middle	54.24	0.00	0.80	0.44	0.08	0.08	0.04	0.04	0.00	0.08
451-475	MIX	0.9	bimodal	24.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
476-500	MIX	0.9	bottom	14.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
501-525	MIX	0.9	middle	60.36	0.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00	0.04
Average				**30.67**	**0.01**	**0.22**	**0.22**	**0.05**	**0.04**	**0.07**	**0.05**	**0.02**	**0.05**

For more details:

Pape, T. 2015. “Heuristics and lower bounds for the simple assembly line balancing problem type 1: Overview, computational tests and improvements.” European Journal of Operational Research 240 (1): 32–42.

Morrison, D. R.; Sewell E. C.; Jacobson S. H. 2014. “An application of the branch, bound, and remember algorithm to a new simple assembly line balancing dataset.” European Journal of Operational Research 236 (2): 403–409.

Computational Results - SALBP-I

Table : Results for the SALBP-I on 1000-items instances. Deviation to the best known lower bound.

Instance	Struct.	OS	Distrib.	BLB*	BUB	Beam-ACO		SALOME		Tabu-Simple			t-bounded-DP			BBR	**ILS**
						180sec	900sec	180sec	900sec	180sec	900sec	3600sec	180sec	900sec	3600sec	3600sec	300sec
1-25	BN	0.2	bottom	136.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
26-50	BN	0.2	middle	511.32	0.96	16.12	13.84	6.56	6.48	2.08	1.28	1.00	21.00	19.48	18.00	13.36	2.20
51-75	BN	0.2	bimodal	225.36	0.00	0.00	0.00	0.00	0.00	0.16	0.12	0.12	0.00	0.00	0.00	0.00	0.12
76-100	BN	0.6	bottom	137.68	0.00	0.00	0.00	0.00	0.00	0.16	0.08	0.04	0.00	0.00	0.00	0.00	0.00
101-125	BN	0.6	middle	511.12	1.56	21.32	19.84	9.72	9.72	3.72	2.04	1.56	23.92	21.16	18.60	15.16	3.44
126-150	BN	0.6	bimodal	221.52	0.00	0.00	0.00	0.00	0.00	0.80	0.40	0.40	0.00	0.00	0.00	0.00	0.44
151-175	CH	0.2	bottom	137.28	0.00	0.00	0.00	0.00	0.00	0.04	0.04	0.04	0.00	0.00	0.00	0.00	0.00
176-200	CH	0.2	middle	510.28	2.52	22.40	21.44	11.60	11.52	4.28	2.80	2.52	22.88	21.80	18.92	16.68	6.32
201-225	CH	0.2	bimodal	225.88	0.00	0.00	0.00	0.08	0.04	0.36	0.24	0.24	0.00	0.00	0.00	0.00	0.36
226-250	CH	0.6	bottom	136.04	0.00	0.00	0.00	0.00	0.00	0.40	0.20	0.04	0.00	0.00	0.00	0.00	0.00
251-275	CH	0.6	middle	511.16	6.92	28.08	27.24	21.36	21.28	10.00	7.88	6.92	22.96	19.96	17.32	15.36	12.28
276-300	CH	0.6	bimodal	221.20	0.00	0.04	0.04	0.08	0.00	1.28	0.64	0.64	0.00	0.00	0.00	0.00	0.68
301-325	MIX	0.2	bottom	138.04	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
326-350	MIX	0.2	middle	510.84	1.48	17.64	15.88	7.72	7.68	2.56	1.80	1.56	20.64	19.40	17.08	14.36	2.80
351-375	MIX	0.2	bimodal	222.48	0.00	0.00	0.00	0.04	0.04	0.40	0.16	0.12	0.00	0.00	0.00	0.00	0.20
376-400	MIX	0.6	bottom	136.52	0.00	0.00	0.00	0.00	0.00	0.24	0.12	0.08	0.00	0.00	0.00	0.00	0.00
401-425	MIX	0.6	middle	514.60	4.20	25.80	24.60	15.16	15.12	6.52	4.84	4.32	22.92	21.00	17.20	14.84	7.80
426-450	MIX	0.6	bimodal	224.72	0.00	0.04	0.04	0.00	0.00	1.00	0.48	0.24	0.04	0.00	0.00	0.00	0.20
451-475	MIX	0.9	bottom	136.40	0.00	0.08	0.04	0.04	0.04	1.12	0.76	0.56	0.00	0.00	0.00	0.00	0.20
476-500	MIX	0.9	middle	536.00	10.64	37.96	36.40	29.64	29.52	52.40	50.40	32.84	16.00	13.84	10.88	18.84	14.28
501-525	MIX	0.9	bimodal	223.08	0.00	0.52	0.52	2.40	1.96	2.96	1.68	1.44	0.00	0.00	0.00	0.00	1.72
Average				**291.80**	**1.35**	**8.10**	**7.61**	**4.97**	**4.92**	**4.31**	**3.62**	**2.60**	**7.16**	**6.51**	**5.62**	**5.17**	**2.53**

For more details:

Pape, T. 2015. "Heuristics and lower bounds for the simple assembly line balancing problem type 1: Overview, computational tests and improvements." European Journal of Operational Research 240 (1): 32–42.

Morrison, D. R.; Sewell E. C.; Jacobson S. H. 2014. "An application of the branch, bound, and remember algorithm to a new simple assembly line balancing dataset." European Journal of Operational Research 236 (2): 403–409.

Computational Results - BPP-P

Table : Results for the BPP-P on 100-items instances. Comparison with Pereira (2016)

Instance	Structure	OS	Distrib.	Root node/DP Heur.			Enumeration				ILS (600s)			
				Gap	T(s)	#Opt	Gap	T(s)	#Opt	$\#Opt_2^*$	Gap	T(s)	T_{inc}(s)	#Opt
1-25	BN	0.2	bimodal	0	7.92	25	0	0	25	25	**0**	3.75	3.74	**25**
26-50	BN	0.2	bottom	0.27	7.51	24	0	0.03	25	25	0.53	79.40	7.39	23
51-75	BN	0.2	middle	1.4	23.82	11	0.74	2,745.03	16	13	**0.23**	173.05	6.29	**22**
76-100	BN	0.6	bimodal	0.68	5.51	21	0	24.21	25	25	**0.17**	121.09	25.08	**24**
101-125	BN	0.6	bottom	0	2.29	25	0	0	25	25	**0**	24.06	0.00	**25**
126-150	BN	0.6	middle	1.11	6.61	13	0	61.77	25	24	**0**	480.07	15.08	**25**
151-175	CH	0.2	bimodal	0.54	5.09	22	0	0	25	25	**0.00**	125.4	29.39	**25**
176-200	CH	0.2	bottom	0	5.42	25	0	0	25	25	**0**	24.10	0.09	**25**
201-225	CH	0.2	middle	1.57	19.24	6	0.54	2,563.51	18	14	**0.47**	251.28	11.35	**19**
226-250	CH	0.6	bimodal	2.35	1.28	12	0	0.11	25	25	0.32	482.69	10.02	**23**
251-275	CH	0.6	bottom	0.84	0.52	23	0	0	25	25	**0**	48.09	0.00	**25**
276-300	CH	0.6	middle	2.12	3.81	5	0	31.02	25	25	0.36	576.54	42.65	**20**
301-325	MIX	0.2	bimodal	0	9.08	25	0	0	25	25	0.18	61.09	37.08	24
326-350	MIX	0.2	bottom	0	6.64	25	0	0	25	25	1.07	103.39	7.39	21
351-375	MIX	0.2	middle	1.72	23.88	10	0.66	2,288.73	18	15	**0.30**	222.2	6.27	**21**
376-400	MIX	0.6	bimodal	1.55	1.57	16	0	0.01	25	25	**0**	285.52	21.94	**25**
401-425	MIX	0.6	bottom	0.63	0.56	23	0	0	25	25	**0**	72.04	0.00	**25**
426-450	MIX	0.6	middle	1.38	4.27	12	0	44.05	25	25	0.37	600.00	42.55	**20**
451-475	MIX	0.9	bimodal	0.79	0.08	19	0	0	25	25	**0**	600.00	0.00	**25**
476-500	MIX	0.9	bottom	0	0.04	25	0	0	25	25	**0**	0.01	0.00	**25**
501-525	MIX	0.9	middle	1.54	0.31	6	0	0.01	25	25	**0**	600.01	1.18	**25**
Average/Total				0.88	6.45	373	0.09	369.45	502	491	**0.19**	234.94	12.74	**492**

*600 seconds time limit.

**Bold values: ILS results better or equal to Dynamic Programming heuristic (Pereira, 2016)

***Bold and underlined values: ILS results better or equal to Enumeration algorithm (Pereira, 2016).

For more details:

Pereira, J. 2016. "Procedures for the bin packing problem with precedence constraints." European Journal of Operational Research 250 (3): 794–806.

Computational Results - BPPGP

Table : Results for different class of problems (TL = 300s).

Problem	n	CPLEX/First Fit				ILS			
		#opt.	Gap(%)	Dev.	T(s)	#opt.	Gap(%)	Dev.	T(s)
SALB	20	525	0.00	0.00	0.09	525	0.00	0.00	29.16
	50	482	0.89	0.26	53.86	523	0.03	0.00	59.78
	100	306	3.85	2.14	133.37	497	0.13	0.05	80.02
	1000	0	5.45	25.28	301.29	272	0.53	2.54	152.68
BPP-P	20	525	0.00	0.00	0.05	525	0.00	0.00	55.44
	50	457	1.22	0.38	55.10	498	0.19	0.05	118.90
	100	314	4.56	2.63	125.17	426	0.72	0.35	120.33
	1000	6	6.80	29.14	300.33	150	1.32	4.32	233.86
BPPGP-01	20	525	0.00	0.00	0.11	525	0.00	0.00	46.87
	50	442	1.29	0.39	56.43	481	0.33	0.09	76.61
	100	307	4.48	2.54	128.44	398	0.80	0.41	98.64
	1000	0	6.92	31.84	301.31	189	1.90	8.26	211.84
BPPGP-03	20	525	0.00	0.00	0.00	525	0.00	0.00	65.73
	50	484	0.75	0.23	30.02	501	0.16	0.05	85.61
	100	364	4.07	2.53	93.13	406	0.59	0.31	108.72
	1000	41	8.03	35.98	282.19	51	3.62	12.51	278.82

*Gap computed as $100(UB - LB)/UB$ and Deviation computed as $(UB - LB)$, where LB is the best know lower bound.

Outline

1 Introduction

2 Mathematical Formulation

3 Proposed Algorithm

4 Computational Results

5 Conclusion

Conclusion

- Simple algorithm
- Satisfactory results
- Future works:
  - Development of new lower bounds procedures
  - Improving the heuristic
  - Exact approaches

Questions?

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An ILS algorithm for the bin packing problem with generalized precedence constraints

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